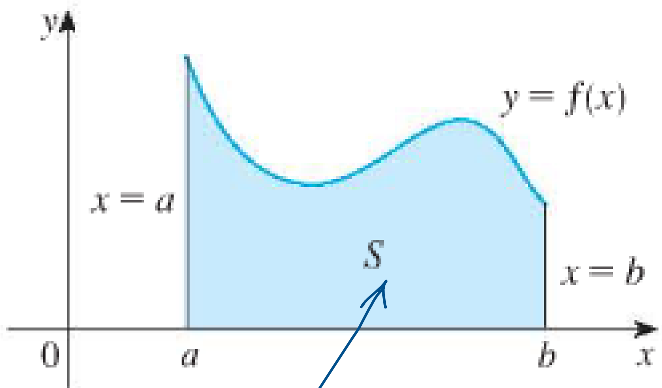
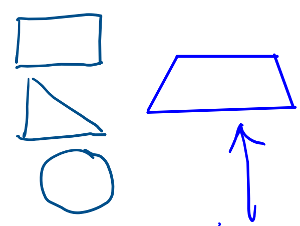
Recall: what does f **prime** tell us about the function f? rate of change / slope of tangent line

The **antiderivative** or $\boxed{\textbf{integral}}$ of f represents <u>bounded area under</u>. curve

The "Problem Child"



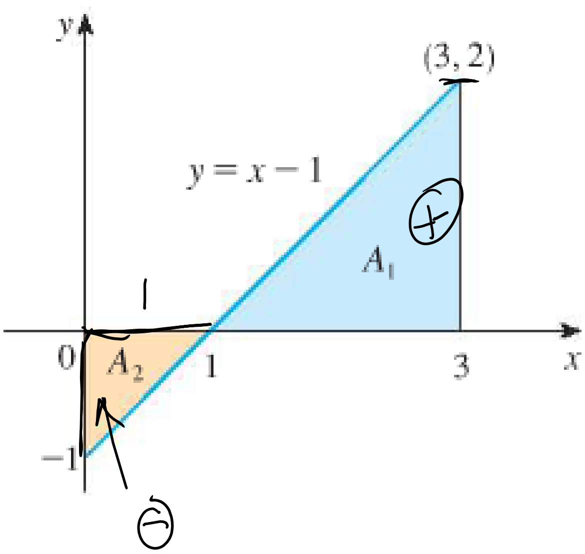


trapezoid

It's a challenge to determine the area under "non-standard" shapes such as the bounded area under a function curve.

If the area under curve is a STANDARD shape then known area formulas can be used.

Caveat: If area lies **BELOW** x-axis, that region is **negative** (subtracted).



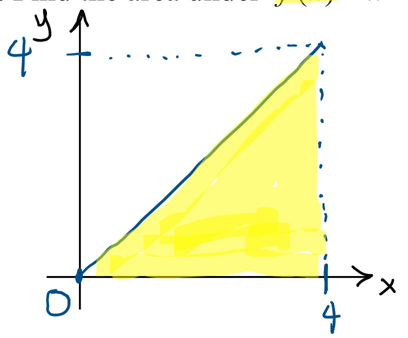
$$A_1 = \tfrac{1}{2}(2)(2) = 2$$

$$A_2 = \tfrac{1}{2}(1)(1) = \tfrac{1}{2}$$

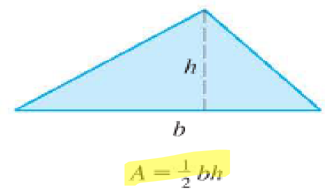
Total area of shaded regions: $A_1 - A_2$

$$2 - \tfrac{1}{2} = \tfrac{4}{2} - \tfrac{1}{2} = \boxed{\tfrac{3}{2}}$$

ex. Find the area under $f(x)=x$ from $\boxed{x=0 \text{ to } x=4}$. ($a$, b)

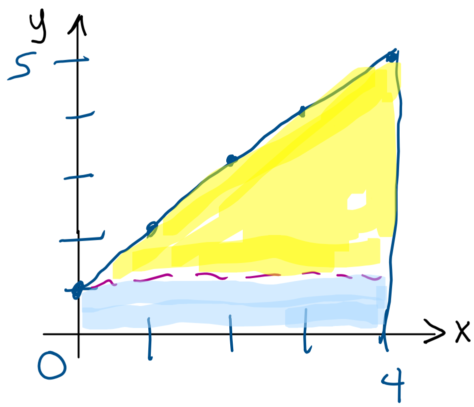


$$A = \tfrac{1}{2} b h = \tfrac{1}{2}(4)(4) = \tfrac{1}{2}\cdot 16 = \boxed{8}$$

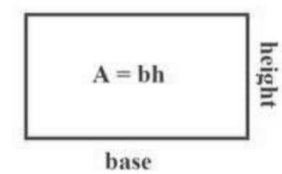


ex. Find the area under $f(x)=x+1$ from $x=0$ to $x=4$. (a, b)

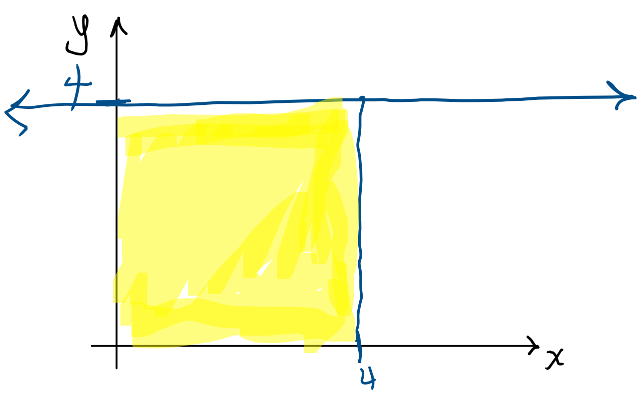
$f(4)=4+1=5$



$$\text{TOTAL AREA} = A_{\triangle} + A_{\square} = \tfrac{1}{2}(4)(4) + (4)(1) = 8 + 4 = \boxed{12}$$



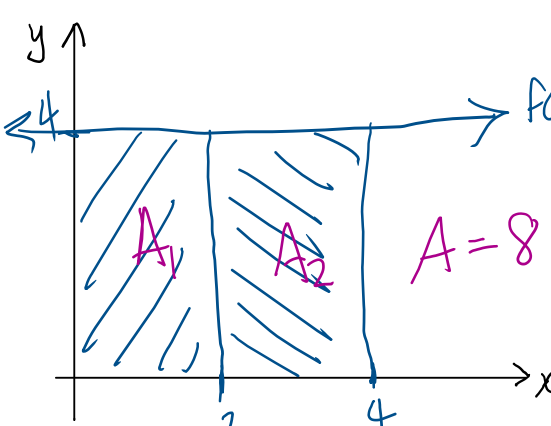
DO: Find the area under $f(x)=4$ from $x=0$ to $x=4$.



$f(x)=4$

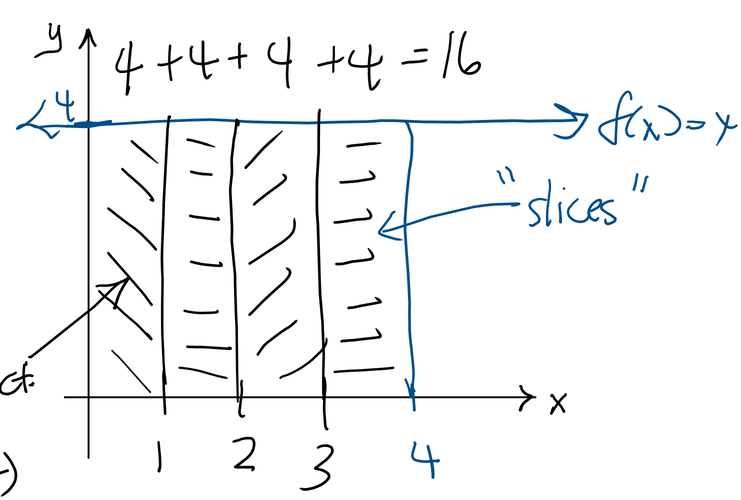
$$A = bh = 4(4) = \boxed{16}$$

Now, let's look at it differently:



$A = 8 + 8 = 16$

$A_1 = 2(4) = 8$ $A_2 = 2(4) = 8$



$4+4+4+4=16$

"slices"

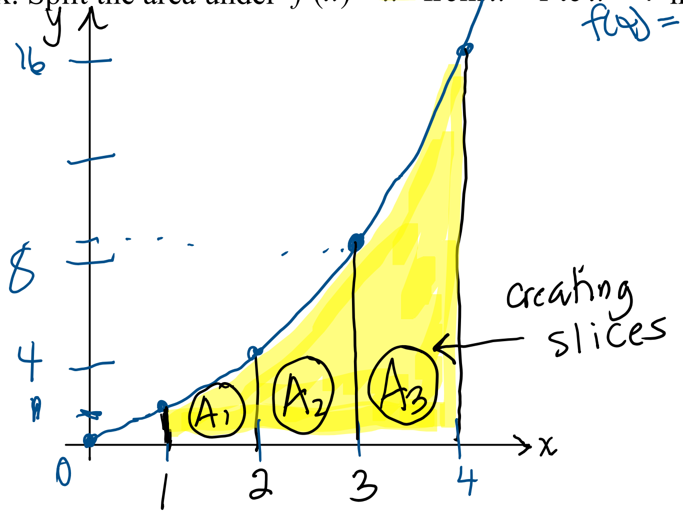
A rect $= bh = 1(4) = 4$

Now, let's apply the rectangle method to **ESTIMATE** the area under parabolas.

ex. Split the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ into 3 regions.

$a \quad b$

$f(4) = 4^2 = 16$



TOTAL AREA: $A_1 + A_2 + A_3$

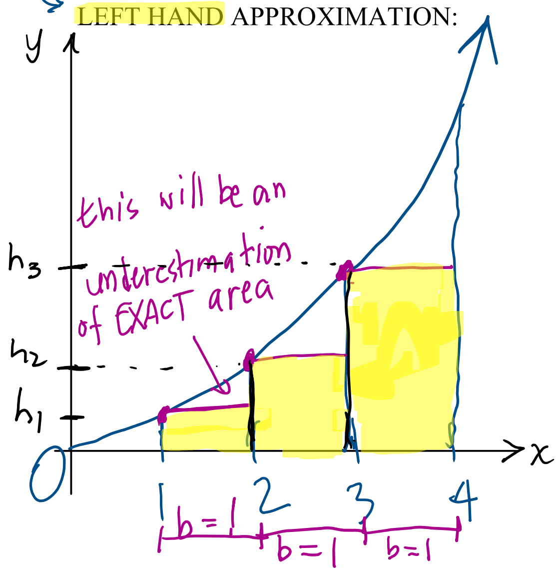
For now, use rectangles to ESTIMATE the exact area under curve:

ex. ESTIMATE the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ *using 3 approximating rectangles.*

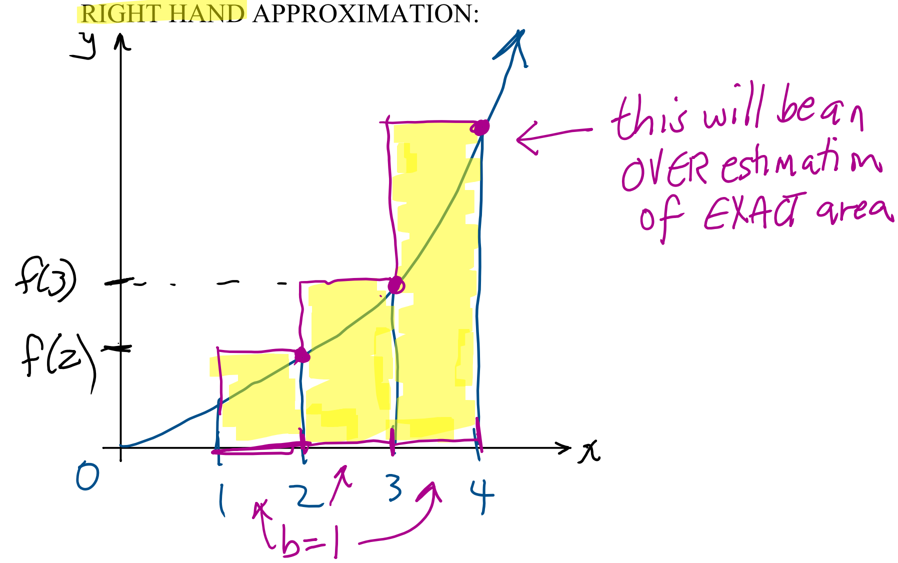
$a \quad b$

using left endpoints

LEFT HAND APPROXIMATION:



RIGHT HAND APPROXIMATION:



$$L_3 = bh_1 + bh_2 + bh_3$$
$$= 1f(1) + 1f(2) + 1f(3)$$
$$= 1^2 + 2^2 + 3^2$$
$$= 1 + 4 + 9 = \boxed{14}$$

$$R_3 = bh_1 + bh_2 + bh_3$$
$$= 1f(2) + 1f(3) + 1f(4)$$
$$= 4 + 9 + 16 = \boxed{29}$$

$$14 < \text{ACTUAL AREA} < 29$$

Computation Efficiency Tip: factor out base

since base is same for all rectangles

$$L_3 = b\underbrace{(h_1 + h_2 + h_3)}_{\text{sum of heights}}$$

n

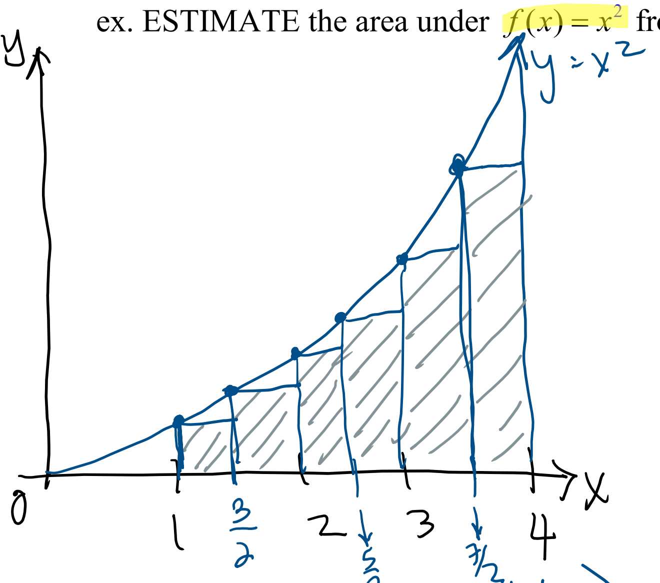
Approximating the area under a curve using the sum of areas of rectangles is called **Riemann Sum**.

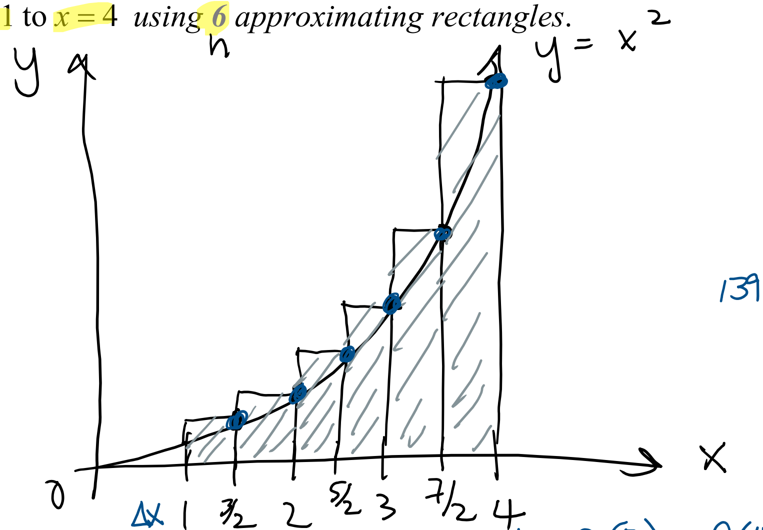
$$\boxed{\Delta x = \frac{b-a}{n}}$$ where a = lower x-bound, b = upper x-bound, n = # of rectangles

Note: bases of rectangles Δx must be the SAME

$$\Delta x = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$$

ex. ESTIMATE the area under $f(x) = x^2$ from $x = 1$ (a) to $x = 4$ (b) *using 6 (n) approximating rectangles*.





$$L_6 = \Delta x\,(\text{sum of heights})$$

$$\text{base} \quad = \frac{1}{2}\left(f(1)+f\left(\tfrac{3}{2}\right)+f(2)+f\left(\tfrac{5}{2}\right)+f(3)+f\left(\tfrac{7}{2}\right)\right)$$

$$= \frac{1}{2}\left(1^2+\left(\tfrac{3}{2}\right)^2+2^2+\left(\tfrac{5}{2}\right)^2+3^2+\left(\tfrac{7}{2}\right)^2\right)$$

$$= \frac{1}{2}\left(1+\frac{9}{4}+4+\frac{25}{4}+9+\frac{49}{4}\right)$$

$$14 \times 4 = 56$$

$$= \frac{1}{2}\left(14\cdot\frac{4}{4}+\frac{9+25+49}{4}\right)$$

$$= \frac{1}{2}\cdot\frac{139}{4} = \boxed{\frac{139}{8}}$$

$$R_6 = \frac{1}{2}\left(f\left(\tfrac{3}{2}\right)+f(2)+f\left(\tfrac{5}{2}\right)+f(3)+f\left(\tfrac{7}{2}\right)+f(4)\right)$$

$$= \frac{1}{2}\left(\frac{9}{4}+4+\frac{25}{4}+9+\frac{49}{4}+16\frac{4}{4}\right)$$

$$139-4=135, \quad +64, \quad 199$$

$$= \frac{1}{2}\cdot\frac{199}{4} = \boxed{\frac{199}{8}}$$

NOTE: using more boxes increases the accuracy of the estimate

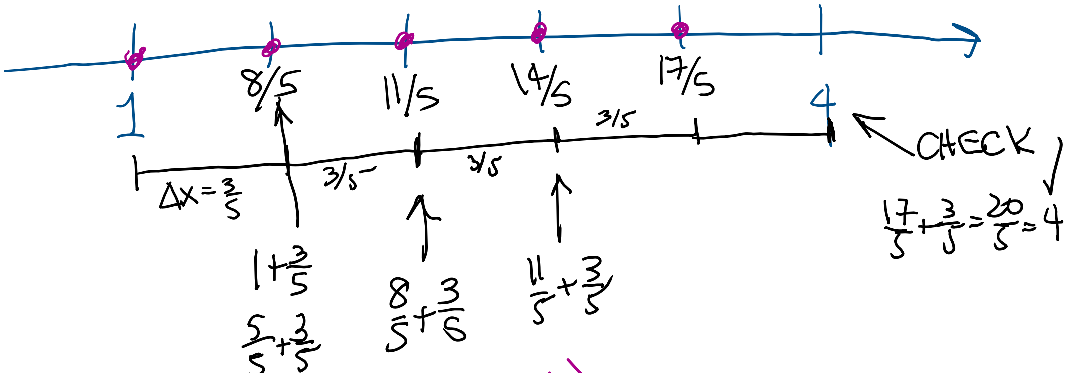
Now, let's determine x-values differently:

ex. SET UP the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ *using* 5 *LEFT-HAND approximating rectangles*.

a b n

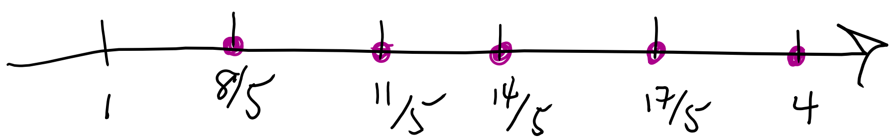
$$\Delta x = \frac{b-a}{n} = \frac{4-1}{5} = \frac{3}{5}$$

find x-values starting at $x = a$



$$L_5 = \overset{\Delta x}{\frac{3}{5}}\left(f(1) + f\left(\frac{8}{5}\right) + f\left(\frac{11}{5}\right) + f\left(\frac{14}{5}\right) + f\left(\frac{17}{5}\right)\right)$$

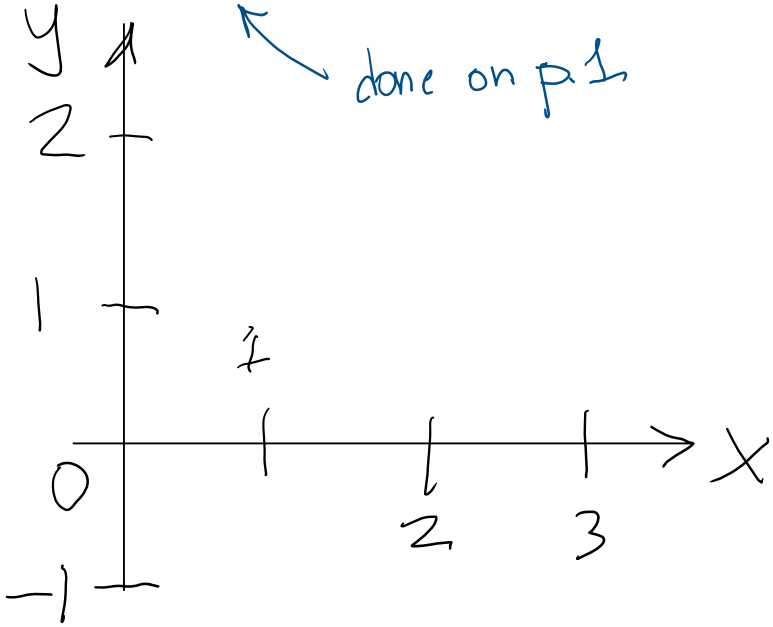
ex. SET UP the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ *using* 5 *RIGHT-HAND approximating rectangles*.



$$R_3 = \overset{\Delta x}{\frac{3}{5}}\left(f\left(\frac{8}{5}\right) + f\left(\frac{11}{5}\right) + f\left(\frac{14}{5}\right) + f\left(\frac{17}{5}\right) + f(4)\right)$$

Revisit: EXACT area under $f(x)=x-1$ from $x=0$ to $x=3$.

done on p.1



ex. SET UP the area under $f(x)=\sin x$ from $x=0$ to $x=\frac{3\pi}{2}$ *using* ***6*** *LEFT-HAND approximating rectangles*.

next time :)